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Stability of Simply Supported Smart Piezolaminated Composite Plates using Finite Element Method

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***Abstract*—Piezolaminated smart structures are mostly used as light weight structures to control structural response in various structural applications. Piezoelectric materials possesses a property of direct and converse piezoelectric effects which can be adequately employed to control the deflection, vibration, shape and buckling of the structure. Due to the application of piezoelectric materials to control structural response, stability and control of light weight structures tends to be the governing criterion which requires significant attention. A finite element methodology is developed for stability analysis of smart piezolaminated composite plates subjected to combined action of electrical and mechanical loading. The finite element formulation is based on higher order shear deformation theory. Numerical analysis is made for stability analysis of simply supported piezolaminated plate.**

***Keywords*—Piezoelectric, Finite element method, vibration and stability.**

I. Introduction

Piezolaminated smart structures are becoming more popular as they can be used as light weight structures to control structural response in various structural applications. Piezoelectric materials have direct and converse piezoelectric effects which can be adequately employed to control the deflection, vibration, shape and buckling of the structure. Due to the application of piezoelectric materials to control structural response vibration, stability and control of light weight structures tends to be the governing criterion which requires significant attention. A finite element methodology is developed for analysis of smart piezolaminated composite plates subjected to combined action of electrical and mechanical loading. The finite element formulation is based on higher order shear deformation theory. Numerical analysis is carried out for stability analysis of simply supported piezolaminated plates.

Smart material systems and structures have been adopted by engineering and research community over the last two decades. In the recent past theoretical as well as experimental investigations has been successfully carried out in the area of smart structures. The need of light weight structures in engineering applications has led to the gradual replacement of many isotropic materials with composites which provide both high stiffness and low weight. Additional requirements for multi functionality, active vibration, noise control and structural health monitoring has led to the development of adaptive piezolaminated materials and structures. The coupled electromechanical properties of piezoelectric ceramics and their availability in the form of thin sheet makes them well suited for use as distributed sensors and actuators for controlling structural response.

modeling and shape control of composite beams with embedded piezoelectric actuators is carried out considering piezoelectric effect (Dhonthireddy and Chandrashekhara, 1996), whereas shape control of non-symmetric piezolaminated composite beams has been perormed for the same (Eisenberger and Abramovich, 1997). Nguyen et al. (2007) presented evolutionary piezoelectric actuators design optimization for static shape control of smart plates. Analysis of piezolaminated beams with large deformations is available (Mukherjee and Chaudhuri, 2002). Large deformation analysis of piezolaminated smart structures using higher-order shear deformation theory is provided further (Kulkarni and Bajoria, 2007). Some work on postbuckling and vibration characteristics of piezolaminated composite plate subject to thermo-piezoelectric loads has been carried out (Oh and Lee, 2000). Postbuckling of shear deformable laminated plates with piezoelectric actuators under complex loading condition including thermo-electro-mechanical loading is conducted further (Shen, 2001). Thus in present work the stability problem of piezolaminated plates/shells is examined considering electromechanical loading. In the present work vibration and stability of piezolaminated plates is studied subjected to combined action of electrical and mechanical loading. The efficiency of finite element model developed is verified by comparing the results obtained with those of available in literature and found to be in good agreement.

II. Finite Element Formulation

It is observed that most of the theories for analysis of plates and shells are based on the classical and first order shear deformation theory, which requires shear correction factor. Whereas higher order shear deformation theory assume realistic cross section deformation pattern through the shell thickness. Here a finite element formulation for stability analysis of piezolaminated smart plate / shell is based on higher order shear deformation theory.

Fig. 1 shows a laminated composite plate provided with piezoelectric patches at top and bottom surface. In higher order shear deformation theory, higher order terms of displacement field are incorporated to consider transverse shear deformation correctly. Thus the three dimensional elasticity problem is approximated to two dimensional formulation using Taylor series expansion for displacement components u, v and w.

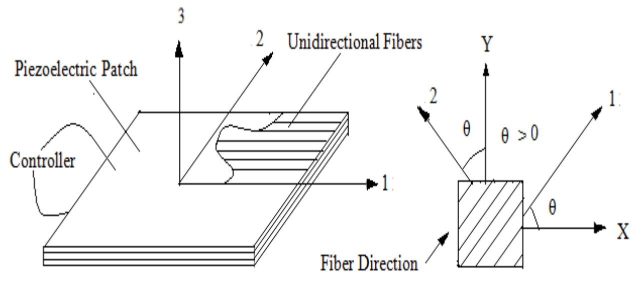

Figure1 Laminated composite plate provided with piezoelectric patches at top and bottom surface.

To account for thickness of laminate, these displacement components are expressed in terms of thickness co-ordinate z. Hence the warping of the transverse cross-section is automatically incorporated. The displacement field is expressed as:

Khare et al (2003) formulated higher order theory for cross ply laminated shallow shells which assumed realistic cross section deformation pattern through the thickness. Thus the displacement field is assumed as:
Assuming the condition of zero shear stresses at the top and bottom surface of laminate, equations for HOST 9 reduced to

$$u = u_0 + z\theta_x + z^2 u_0^* + z^3 \theta_x^*$$
$$v = v_0 + z\theta_y + z^2 v_0^* + z^3 \theta_y^*$$
$$w = w_0 \qquad (1)$$

Where, $u(x,y,z)$, $v(x,y,z)$ and $w(x,y,z)$ are the displacement of any point in the plate domain in in x, y and z direction respectively. $u_0(x,y,z)$, $v_0(x,y,z)$ and $w_0(x,y,z)$ are the displacement of midpoint of normal. $\theta_x(x,y)$, $\theta_y(x,y)$ are the rotations of normal at the middle plane in x and y direction about y and x axis respectively. $u_0^*(x,y)$, $v_0^*(x,y)$, $w_0^*(x,y)$, $\theta_x^*(x,y)$ and $\theta_y^*(x,y)$ are higher order terms which accounts cubic variation of normal.

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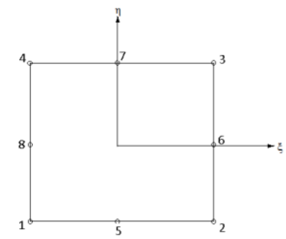
Strain-Displacement Relations

The strains associated with the displacement model for linear bending are given by

$$\varepsilon_x = \varepsilon_{x0} + z\chi_x + z^2 \varepsilon_{x0}^* + z^3 \chi_x^*$$
$$\varepsilon_y = \varepsilon_{y0} + z\chi_y + z^2 \varepsilon_{y0}^* + z^3 \chi_y^*$$
$$\varepsilon_z = \varepsilon_{z0} + z\chi_z + z^2 \varepsilon_{z0}^* + z^3 \chi_y^*$$
$$\gamma_{xy} = \varepsilon_{xy0} + z\chi_{xy} + z^2 \varepsilon_{xy0}^* + z^3 \chi_{xy}^*$$
$$\gamma_{xz} = \varphi_x + z\chi_{xz} + z^2 \varphi_x^* + z^3 \chi_{xz}^*$$
$$\gamma_{yz} = \varphi_y + z\chi_{yz} + z^2 \varphi_y^* + z^3 \chi_{yz}^* \qquad (2)$$

Shell Element and Shape functions



Parent element

For corner nodes

$$N_i(\xi,\eta) = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(-1+\xi\xi_i+\eta\eta_i),$$

for i = 1,2,3,4

For middle edge node

$$N_5(\xi,\eta) = \frac{1}{2}(1-\xi^2)(1-\eta)$$
$$N_7(\xi,\eta) = \frac{1}{2}(1-\xi^2)(1+\eta)$$
$$N_6(\xi,\eta) = \frac{1}{2}(1+\xi)(1-\eta^2)$$
$$N_8(\xi,\eta) = \frac{1}{2}(1-\xi)(1-\eta^2) \qquad (3)$$

Geometry within the Element and Displacement Field

Fig. 2 shows a typical eight noded degenerated quadrilateral shell element. x, y, z are the global coordinates and ξ, η *and* ζ are the natural coordinates. ζ=0 represents the mid surface. ζ=1 represents outer surface of shell and ζ= -1 represents the inner surface of shell. Coordinates of middle plane points can be obtained as below.

$$x = \frac{xt + xb}{2}, \quad y = \frac{yt + yb}{2}, \quad z = \frac{zt + zb}{2} \tag{4}$$

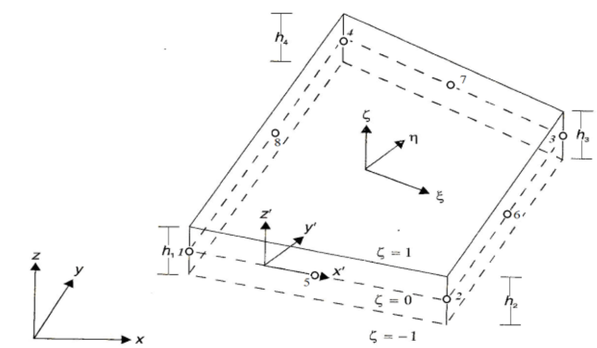


Figure 2 Coordinate system for 8-noded degenerated shell element

The global coordinates of any point in the element at a distance ζ on normal are given by

$$x = \sum_{i=1}^{8} N_i\, x_i + \frac{\varsigma}{2}\sum_{i=1}^{8} N_i\, \Delta x_i$$

$$y = \sum_{i=1}^{8} N_i\, y_i + \frac{\varsigma}{2}\sum_{i=1}^{8} N_i\, \Delta y_i$$

$$z = \sum_{i=1}^{8} N_i z_i + \frac{\varsigma}{2}\sum_{i=1}^{8} N_i \Delta z_i \tag{5}$$

Where, $\Delta x_i = x_{t_i} - x_{b_i}$, $\Delta y_i = y_{t_i} - y_{b_i}$ $\Delta z_i = z_{t_i} - z_{b_i}$ and $N_i(\xi,\eta)$ are shape functions as given in equation (3).
Thickness vector at any node is written as $\overrightarrow{v_{z_i}} = (x_t - x_b)_i + (y_t - y_b)_j + (z_t - z_b)_k$

Displacement field

At each node introduce two vectors $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ mutually perpendicular in the tangential plane. $\bar{v}_{1i,}$ $\bar{v}_{2i}$ and $\bar{v}_{3i}$ are the unit vectors in the direction of local axes, i, j and k are unit vectors in the direction of global axes. Let u, v, w be the displacement of a point having its local coordinates; ui, vi, wi be the displacement of corresponding mid surface which is having local coordinates ξ,η and ς . u_i^*, v_i^*, w_i^* be the relative displacement along global x, y, z directions due to rotation of normal at node i, i.e. $\theta_{xi}, \theta_{yi}, \theta_{zi}$ about the global axes.

$$u = \sum_{i=1}^{8} N_i\, u_i + \sum_{i=1}^{8} N_i\, u_i^* \tag{6}$$

Thus, writing

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{8} Ni \left[\begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \begin{Bmatrix} u_i^* \\ v_i^* \\ w_i^* \end{Bmatrix} \right] \tag{7}$$

$$\bar{v}_{1_i} = l_{1_i}\, i + m_{1_j}\, j + n_{1_k}\, k$$

Where, l_{1_i}, m_{1_j} and n_{1_k} are direction cosines of vector $\bar{v}_{1_i}$.

Similarly,

$$\bar{v}_{2_i} = l_{2_i}\, i + m_{2_j}\, j + n_{2_k}\, k$$

$$\bar{v}_{3_i} = l_{3_i}\, i + m_{3_j}\, j + n_{3_k}\, k \tag{8}$$

For a point which is at middle plane w = 0.

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{8} N_i \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \sum_{i=1}^{8} N_i \begin{bmatrix} l_{1_i} & l_{2_i} \\ m_{1_i} & m_{2_i} \\ n_{1_i} & n_{2_i} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \tag{9}$$

At any point i, for a point located at a distance ς from middle plane will undergo some u, v displacement due to rotation α_1 and α_2.

$$\begin{Bmatrix} \alpha_{1i} \\ \alpha_{2i} \end{Bmatrix} = \begin{bmatrix} l_{1i} & m_{1i} & n_{1i} \\ l_{2i} & m_{2i} & n_{2i} \end{bmatrix} \begin{Bmatrix} \theta_{xi} \\ \theta_{y_i} \\ \theta_{zi} \end{Bmatrix} \tag{10}$$

$\Delta\varsigma = 2 \longrightarrow \Delta x = x_t - x_b = t_i$ (thickness at ith node)
At middle plane

$$\varsigma = \frac{\varsigma}{2} h_i\,; \quad u = \frac{\varsigma}{2} h_i\, \alpha_{1_i}\,; \quad v = -\frac{\varsigma}{2} h_i\, \alpha_{2_i}\,; \; w=0$$

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{8} N_i \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \sum_{i=1}^{8} N_i \begin{bmatrix} l_{1_i} & -l_{2_i} \\ m_{1_i} & -m_{2_i} \\ n_{1_i} & -n_{2_i} \end{bmatrix} \frac{\varsigma h_i}{2} \begin{Bmatrix} \alpha_{1_i} \\ \alpha_{2_i} \end{Bmatrix} \tag{11}$$

Displacement-strain relation

Strains are related with displacements as follows,

$$\underset{\sim}{\varepsilon} = B\, \underbrace{\delta_e} \tag{12}$$

And

$$\underset{\sim}{\gamma} = B_s\, \underbrace{\delta_e} \tag{13}$$

Electro-Mechanical Coupling

Due to the direct and converse piezoelectric effect there exists a coupling between electrical and mechanical loading in smart piezoelectric structures. Thus the piezoelectric equations can be decoupled resulting in electromechanical coupling. Variation of temperature effect is neglected in formulation. The constitutive equations of a piezoelectric material including the effect of electrical and mechanical expansion can be expressed as follows. Hence dielectric displacement vector in local coordinate field is given as, For the piezoelectric layer polarized in the thickness direction, the dielectric displacement vector using direct piezoelectric equation is,

$$\{D\} = [e]\{\varepsilon\} + [g]\{E^p\}$$
$$\{\sigma\} = [C]\{\varepsilon\} - [e]^t\{E^p\} \qquad (14)$$

In all above equations, {D} is electric displacement vector, [e] is dielectric permittivity matrix, ε is the strain vector, $\{g\}$ is the dielectric matrix. $\{E^p\}$ is the electric field vector, $[\sigma]$ is the stress vector and [C] is the elastic matrix for constant electric field.

Electrical Potential Function

One electrical degree of freedom is adopted per node of an element. If ϕ_a *and* ϕ_s are the electric displacement at any point in the actuator and the sensor layers, respectively, the electrical potential functions in terms of the nodal potential vector are given by

$$\phi_a^{'} = [N_{pa}]\{\phi_a^e\}$$
$$\phi_s^{'} = [N_{ps}]\{\phi_s^e\} \qquad (15)$$

Where, $\lfloor N_{pa} \rfloor$ and $\lfloor N_{ps} \rfloor$ are the shape function matrices for the actuator and sensor layers, respectively. $\{\phi_a^e\}$ and $\{\phi_s^e\}$ are the nodal electric potential vector for the actuator and sensor layers, respectively and can be given as follow.

$$\{\phi_a^e\} = \{\phi_{a1}\ \phi_{a2}\ \phi_{a3} \ldots\ldots\ldots \phi_{an}\}^T$$

and

$$\{\phi_s^e\} = \{\phi_{s1}\ \phi_{s2}\ \phi_{s3} \ldots\ldots\ldots \phi_{sn}\}^T \qquad (16)$$

The electric field strength of an element in terms of the electrical potential of the actuator and sensor layers is expressed as

$$\{E_a^P\} = \begin{Bmatrix} E_x^P \\ E_y^P \\ E_z^P \end{Bmatrix}_a = -\begin{Bmatrix} \partial\phi_a/\partial x \\ \partial\phi_a/\partial y \\ \partial\phi_a/\partial z \end{Bmatrix}$$

$$\{E_s^P\} = \begin{Bmatrix} E_x^P \\ E_y^P \\ E_z^P \end{Bmatrix}_s = -\begin{Bmatrix} \partial\phi_s/\partial x \\ \partial\phi_s/\partial y \\ \partial\phi_s/\partial z \end{Bmatrix} \qquad (17)$$

The electric field vector for the actuator and sensor layer can be modified as

$$\{E_a^P\} = -\sum_{i=1}^{n} B_{a(i)}\,\phi_{a(i)}^e = -[B_a]\{\phi_a^e\}$$
$$\{E_s^P\} = -\sum_{i=1}^{n} B_{s(i)}\,\phi_{s(i)}^e = -[B_s]\{\phi_s^e\} \qquad (18)$$

Stiffness Matrix Equations

Element stiffness matrix can be written as,

$$\lfloor K^e \rfloor\{\delta^e\} + \lfloor K_\sigma^e \rfloor\{\delta^e\} = \lfloor F_1^e \rfloor + \lfloor F_{ac}^e \rfloor \qquad (19)$$

In which

$$[K^e] = [K_d^e] + [K_{aa}^e]^{-1}[K_{aa}^e][K_{ad}^e] + [K_{ds}^e][K_{ss}^e]^{-1}[K_{sd}^e] \qquad (20)$$

$$\{F_{ac}^e\} = [K_{da}^e][K_{aa}^e]^{-1}\{Q_a^e\} \qquad (21)$$

Where,

$$[K_d^e] = \int_V [B]^T[C][B]\,dV\ ,$$
$$[K_{da}^e] = [K_{ad}^e]^T = \int_{Va} [B]^T[e][B_a]\,dV\ ,$$
$$[K_{aa}^e] = \int_{Va} [B_a]^T[g][B_a]\,dV\ ,$$
$$[K_{ds}^e] = [K_{sd}^e]^T = \int_{Vs} [B]^T[e][B_s]\,dV \text{ and}$$
$$[K_{ss}^e] = \int_{Vs} [B_s]^T[g][B_s]\,dV \qquad (22)$$

Thus the element equation in the global stiffness matrix can be written as

$$[K]\{\delta\} + \lfloor K_\sigma \rfloor\{\delta\} = \lfloor F_1 \rfloor + \lfloor F_{ac} \rfloor \qquad (23)$$

Stability Criterion

Stability is associated with for the given loading whether response of the structure remains bounded or it goes unbounded. The critical load is the load under which the structure can be in equilibrium both in the straight (initial) and the slightly bent configuration. If λ is a scalar multiplier for any arbitrary reference for which, geometric stiffness matrix, based on an arbitrary reference intensity of membrane streeese. λ is determined such that both the reference configuration represented by the load vector {d} and slightly deformed remains in equilibrium configuration. Thus For free vibration problems, the equations of motion can be expressed as the following eigenvalue problem form which natural frequency can be calculated:

$$\left(\left[K - \omega^2 [M]\right]\right)\{\delta_E\} = \{0\} \qquad (24)$$

Where, matrix [K] denotes the stiffness matrix which may contain the terms of the in-plane stresses and matrix [M], the mass matrix.
For stability problems, the natural frequency vanishes and the stability equation can be expressed as the following eigenvalue problem:

$$\left([K] + \lambda [K_\sigma]\right)\{\delta\} = F \qquad (25)$$

$$\left([K] + \lambda [K_\sigma]\right)\left(\{\delta\} + \{d\delta\}\right) = \{F\} \qquad (26)$$

Where, the lowest magnitude of eigen value gives critical buckling load and the vector $\{d\delta\}$ represents the buckled mode shape.

$$\left([K] + \lambda [K_\sigma]\right)\{d\delta\} = 0 \qquad (27)$$

The buckling stresses can be calculated through the stability Eq. (51) as eigenvalue problems. Another method to obtain the critical buckling stresses of piezolaminated plates subjected to in-plane stresses is to compute natural frequencies by increasing the absolute values of compressive stresses until the lowest natural frequency vanishes. In the case of simply supported cross-ply piezolaminated composite plates subjected to in-plane stresses σ the natural frequency ω_a can be expressed explicitly with reference to the natural frequency ω_0 of laminates without in-plane stresses. The relation between ω_a and ω_0 can be obtained from a comparison of the equations of motion as follows:

$$\omega_a^2 = \omega_0^2\ \pi^2 \left[r^2 + k\left(\frac{a}{b}\right)^2 s^2 \right]\left(\frac{h}{a}\right)\lambda \qquad (28)$$

Thus

$$\lambda_{cr} = -\frac{\omega_0^2}{\pi^2 \left[r^2 + k\left(\frac{a}{b}\right)^2 s^2 \right]\left(\frac{h}{a}\right)^2} \qquad (29)$$

III. Numerical Analysis

Buckling of simply supported square piezoelectric laminated plate (*p*/0^0/90^0/90^0/0^0/*p*)

The piezoelectric laminates with piezolayer attached at the top and bottom of the plate is subjected to a uniaxial uniform edge compressive force N_x. Hence a square piezoelectric laminated plate of thickness of 0.01 m and side length ‘a’ is having simply supported boundary on all four edges. The laminate consists of a (*p*/0^0/90^0/90^0/0^0/*p*) Graphite-Epoxy sublaminate provided with two PZT-5A attached on outer surfaces of the plate. Each piezoelectric layer has thickness of 0.1h, whereas each elastic layer has a thickness of 0.2h.

The effect of electric condition is examined. However, its outer surface may be grounded (closed circuit condition) or remains free (open-circuit condition). In addition, two types of potential distribution on the open-circuited outer surface are assumed.

TABLE I. ELASTIC AND PIEZOELECTRIC PROPERTIE

Elastic Properties

Properties	Graphite-Epoxy	PZT-4
E_{11} (*Gpa*)	181	61.0
E_{22} (*Gpa*)	10.3	61.0
E_{33} (*Gpa*)	10.3	53.2
G_{12} (*GPa*)	7.17	22.6
G_{23} (*GPa*)	2.87	21.1
G_{32} (*GPa*)	7.17	21.1
υ_{12}	0.28	0.35
υ_{23}	0.28	0.38
υ_{32}	0.33	0.38

Piezoelectric Properties

Properties	Graphite-Epoxy	PZT-4
d_{31} (10^{-12} m/V)	0	-171
d_{32} (10^{-12} m/V)	0	-171
d_{33} (10^{-12} m/V)	0	374
d_{15} (10^{-12} m/V)	0	584
d_{24} (10^{-12} m/V)	0	584
ε_{11} (10^{-8} F/m)	0.0031	1.53
ε_{22} (10^{-8} F/m)	0.0027	1.53
ε_{33} (10^{-8} F/m)	0.0027	1.5

TABLE II. BUCKLING LOADS

Critical uniaxial buckling load ($N_x a^2/E_2.h^3$) of simply supported square laminated plate ($p/0^0/90^0/90^0/0^0/p$)

a/h		Closed-circuit	Open-circuit
	Present	8.162	8.430
5	Akhras et. Al [11]	8.181	8.517
	Present	14.810	15.518
10	Akhras et. Al [11]	15.006	15.869
	Present	21.570	21.764
100	Akhras et. Al [11]	20.040	22.294

The results of critical uniaxial buckling loads of laminate with closed and open loop shwos that piezoelectricity has little effect on the buckling load of the laminates with closed-circuits. The incerase in critical buckling load is found to be 3.179 %, 4.56 % and 0.89 % for open loop circuit than that of closed loop for a/h = 5, 10 and 100 respectively.

IV. Conclusions

Piezoelectric actuators and sensors are being used increasingly in structural applications involving shape and vibration and buckling control. With proper selection and placement of piezoelectric actuators, it is feasible to generate enough forces on a structure in order to control its response in buckling. It is observed that the maximum percentage variation in critical uniaxial buckling load is 4.56 % for PZT-5 for a/h = 10. Hence the increase in stiffness due to piezoeffect can be considered with sufficient accuracy to control its response.

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