International Journal of Advancements in Mechanical and Aeronautical Engineering – IJAMAE
Volume 1 : Issue 1
Publication Date : 15 May 2013

Fuzzy Sliding Mode Control Design for Synchronization of Chaotic System

Budiman Azzali Basir
Faculty of Engineering & Technology
Multimedia University
Ayer Keroh, Melaka, Malaysia

Mohammad Noh Ahmad and Abdul Rashid Hussain
Faculty of Electrical Engineering
University Technology Malaysia
Skudai, Johor DT, Malaysia

Abstract—**This paper presents the studied of sliding mode control for a class of chaotic system via Takagi Sugeno fuzzy model approach. The method utilized the concept of state feedback control and Lyapunov functional approach to determine the condition of asymptotically stability. The condition for asymptotically stability and the reaching condition are presented in the forms of linear matrix inequalities (LMI), thus the global asymptotically stability of fuzzy sliding mode control via Takagi Sugeno fuzzy model approach for a class of chaotic system is determined. An example is presented to shows the feasibility and the functionality of the proposed method.**

Keywords— Chaotic System, Sliding Mode Control, T-S Fuzzy Model, LMI.

I. Introduction

In recent years, Takagi-Sugeno (TS) fuzzy model based controller design have attracted and gained much success in dealing with nonlinearity in system [1]-[3]. Furthermore the stabilization issues in the early of Takagi-Sugeno (TS) fuzzy model approached have been well studied and improved over the years [4]-[15]. In addition, due to the bounded region of the state space govern by the chaotic system has simplified the works of representing the system by T-S fuzzy model.

Following the synchronization of two chaotic dynamical systems by Pecora and Caroll [16], have created a numerous attention in dealing with synchronization of chaos system. As a result, a variety of control methods have been proposed [17]-[19]. For instance, a few approach the designing parameter need to use a high gains while other need to satisfies the nonlinear terms using Lipschitz condition.

Research in variable structure control offers an alternative method in dealing with system uncertainties. This statement can be supported by the numbers of research over the last few decades in sliding mode control (SMC) theory [20]-[23]. Furthermore the variable structure system can catered the nonlinearity in system parameter with much affects. However there is a setback in sliding mode control approach which is the chattering issue. Many researchers have tried to alleviate the chattering of control [24]. Therefore, we try to alleviate the chattering in sliding mode control of chaotic system by utilizing the fuzzy model based approach. In this paper, we utilized the concept of state feedback and Lyapunov functional approach to obtain a sufficient stability condition for designing a robust sliding mode plane. Then the system stability issues are formulated into sets of Linear Matrix Inequalities (LMI). A numerical example of a chaotic is simulated using the proposed algorithms to show the effectiveness and the feasibility of the controller.

This paper is organized as follows. Section II presents the Takagi Sugeno fuzzy model based of chaotic system. Section III presents the fuzzy sliding mode control design. Section IV presents the numerical example of the control approach with the simulation results and analysis. Lastly, the conclusion is briefed in the section V.

II. Takagi Sugeno Fuzzy Model Based of Chaotic System.

The fuzzy model proposed by Takagi and Sugeno [1] is described by IF-THEN rules, which represent the local linear input output relations as the consequent parts. In order to design a T-S fuzzy model based synchronization of chaotic system, the chaotic system itself must be represented in T-S fuzzy models.

Consider a general chaotic dynamical system as follows:

$$sx(t) = f(x(t)) + g(x(t)u(t) + C \qquad (1)$$

where $sx(t)$ is $\dot{x}(t)$ in continuous time and $x(t)$ and $u(t)$ are the state and input vector respectively, C is the constant vector while $f(\cdot)$ and $g(\cdot)$ are the nonlinear function with appropriate matrices dimensions. Therefore the T-S fuzzy model can be represented as follows:

Model Rule i: IF $z_i(t)$ is M_i^1 and … and $z_p(t)$ is M_p^i
THEN $\dot{x}(t) = A_i x(t) + B_i u(t) + f_i(x,t) \qquad (2)$

where $i = 1,2 \ldots, r$. r is the number of IF-THEN rules and $x(t) \in R^n$ is the state vector of the systems. $u(t) \in R^m$ and $y(t) \in R^q$ is the input vector and the output vector of the systems respectively. $B_i \in R^{n\times m}$ and $A_i \in R^{n\times n}$ are the system input matrices and the systems matrices and respectively. ΔA_i represents the parameters uncertainties and $f_i(x,t)$ is bounded external disturbance. $M_j^i, j = 1,2 \ldots, p$ is denoted as the j^{ith} fuzzy set for the i^{ith} rule and $z_1(t), .., z_p(t)$ are the known variables functions of state variables. $w_j^i(z_j)$ is denoted as the membership function for j^{ith} fuzzy set M_j^i of i^{ith} and $w_i(z(t) = \prod_{j=1}^{p} M_j^i\,(z(t))$ $i = 1, .., r$

Let's denotes the pair of $x(t), u(t)$, as the fuzzy systems output represented as [15]:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(t)\left((A_i x(t)) + B_i u(t) + f_i(x,t)\right)}{\sum_{i=1}^{r} w_i(t)} \quad (3)$$

$z(t)$ is the premise vector for $z(t) = z_1, z_2 \cdots z_p$

As the $z(t)$ is regards as the combination of linear and state vector, thus the weight function can be represented as:

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(t)} \quad , i = 1, \cdots, r \quad (4)$$

for all t.

$w_j^i(z_j)$ denoted as the membership grade for $z_j(t)$ in M_j^i from (2), noting

$$\begin{cases} \sum_{i=1}^{r} h_i(z(t)) = 1 \\ h_i(z(t)) \geq 0 \end{cases} \quad i = 1, \cdots, r$$

Therefore the Takagi Sugeno fuzzy model is represents as:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(t)\left((A_i x(t)) + B_i u(t) + f_i(x,t)\right) \quad (5)$$

and the output of Takagi Sugeno fuzzy model is represents as:

$$y(t) = \sum_{i=1}^{r} h_i(t)\, C_i\, x(t) \quad (6)$$

III. Fuzzy Sliding Mode Controller Design

The studied of derived sliding plane using Lyapunov functional approach for SMC theory has been studied in [20]. Therefore we applied this concept of approach via Takagi Sugeno fuzzy model for a class of nonlinear systems.

Before proceeding, the following assumptions are needed.

Assumption 1: There exists $K \in R^{m\times n}$ for the pair (A_i, B) is potentially stabilize such that $\bar{A}_i = A_i - BK$ is stable.

Assumption 2: $B_1 = B_2 = \cdots = B_n := B$ and B is a full column rank matrices.

Assumption 3: The external disturbances satisfying

$$f_i(x,t) = B\bar{f}_i(x,t), \quad (7)$$

$$\left\|\bar{f}_i(x,t)\right\| \leq \delta(t), \quad (8)$$

The first step is to determine the plane for the sliding mode in order to design the fuzzy sliding mode control Therefore we choose the sliding plane to be:

$$S = B^T P x(t) \quad (9)$$

where $P \in R^{m\times n}$ is a determined positive definite matrices.

The main purpose of this section is to predetermine the stability condition for the nonlinear Takagi Sugeno fuzzy model in two parts. The first part is to derive an appropriate sliding plane that guaranteed the trajectories of the systems at any given initial states values can converge into sliding plane within finite time. Second part is to determine the conditions that is sufficient to achieve an asymptotically stability of the control system.

Theorem 1: Noting on the assumptions 1-4, the trajectories for a class of nonlinear system at any given initial states, are brought within the sliding plane within a finite time via Takagi Sugeno fuzzy model with a given control as:

$$u(t) = u_{eq} + u_n \quad (10)$$

where u_{eq} is the equivalent control described as:

$$u_{eq} = -\sum_{i=1}^{n} h_i(z(t))(B^T PB)^{-1}[B^T P A_i x(t)] \quad (11)$$

and u_n is the switching control described as:

$$u_n = -\sum_{i=1}^{n} h_i(z(t))\{(B^T PB)^{-1}[\|B^T PB\| \cdot \delta + \varepsilon_0] sgn(s)\} \quad (12)$$

where ε_0 is a small positive constant.

Proof: Consider the Lyapunov function

$$V = 0.5 S^T S \quad (13)$$

Thus derivative of the Lyapunov functional (13) along the trajectory of the system (3) is:

$$\begin{aligned} \dot{V} &= S^T \dot{S} = S^T B^T P \dot{x}(t) \\ &= \sum_{i=1}^{n} h_i(z(t))\, S^T B^T P[A_i x(t) + B_i u(t) + f_i(x,t)] \end{aligned}$$

Substituting (13) into the above equation, yields

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n} h_i(z(t))\, S^T B^T P[A_i x(t) + B_i u(t) + f_i(x,t)] \\ &= \sum_{i=1}^{n} h_i(z(t))\, [S^T B^T P B \bar{f}_i(x,t) + S^T B^T P B u_n] \end{aligned}$$

Considering (7)-(10) and (4), $\dot{V}$ can be expressed as

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^{n} h_i(z(t))\, \|S^T\| [\|B^T P B_i\| \cdot \delta_f(t)] + S^T B^T B u_n \\ &= \sum_{i=1}^{n} h_i(z(t))\, \varepsilon_0 S^T sgn(S) \\ &= \varepsilon_0 \|S\| \\ &\leq 0 \end{aligned}$$

This proved that the trajectories of the system will converge into sliding plane within a finite time. Therefore the proof is completed.

The subsequent part is to derive a robust switching control so that trajectories of the system will remains in the sliding plane once in reached and maintaining the stability of the system even in the presence of disturbance. Thus the following is obtained.

Theorem 2: [4] The fuzzy model in (2) is said to be asymptotically stable when there exist a common positive definite matrix P which satisfies the following Lyapunov inequalities:

$$G_{ii}^T P + P G_{ii}^T < 0 \quad (14)$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) < 0 \quad \text{for } i < j \quad (15)$$

where $G_{ij} = A_i - B_i F_j$

For each $i, j \in \{1,2 \cdots r\}$ except for pairs of $\forall t\ h_i(z(t)) h_j(z(t)) \neq 0$ where r is the number of if-then rules.

Using linear matrix inequalities (LMI's) transformations [3], yields the exponential stability condition [7]. Then transforms into LMIs term by multiplication of P^{-1}. Denoting $X = P^{-1}$ and $M_i = F_i X$ yields:

$$X A_i^T + A_i X - B_i M_i - M_i^T B_i^T < 0 \qquad i = 1 \cdots r \qquad (16)$$

$$X(A_i^T + A_j^T) + (A_i + A_j)X - (B_i M_j + B_j M_i) - (B_i M_j + B_j M_i +)^T < 0$$
$$i < j \le r \qquad (17)$$

IV. Numerical Example

Consider the Rossler chaotic system given by:

$$\dot{x}_1(t) = -x_2 - x_3$$
$$\dot{x}_2(t) = x_1 - 0.2x_2$$
$$\dot{x}_3(t) = 0.2 + x_3(x_1 - 5) \qquad (18)$$

The system is chaotic without control. This chaotic system is constructed by linearized the nonlinear terms to a set of operation point at phase plane in the form of T-S fuzzy model. Therefore the T-S fuzzy model rules are obtained as follows:

Model Rule 1: IF x_2 is M_1 ,

THEN $\dot{x}(t) = A_1 x(t) + B_1[u(t) + f_1(x,t)]$

Model Rule 2: IF x_2 is M_2 ,

THEN $\dot{x}(t) = A_2 x(t) + B_2[u(t) + f_2(x,t)]$

where

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & 5.5 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & -15.5 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix},$$

The chosen membership functions for $x_1(t)$ are as:

$$F_1 = \frac{1}{2}\left(1 + \frac{x_1}{10.5}\right)$$
$$F_2 = \frac{1}{2}\left(1 - \frac{x_1}{10.5}\right)$$

By using LMI toolbox, solving the LMIs (14)-(17) there exists a feasible solution with the symmetric positive definite P as:

$$P = \begin{bmatrix} 1.2879 & 0.6781 & -0.4892 \\ 0.6781 & 0.8019 & -0.2558 \\ -0.4892 & -0.2558 & 0.3005 \end{bmatrix}$$

Based on the two theorems presented, the given uncertain nonlinear system is stably robust and therefore asymptotically stability is guaranteed.

To alleviate the chattering control effect we substituted $sgn(S)$ with $S/(\|S\| + 0.1)$ as method in [24]. The simulation results with given initial $x(0) = [0.5\ 0.5\ 0.5]^T$ are shown in Figure 1-3.

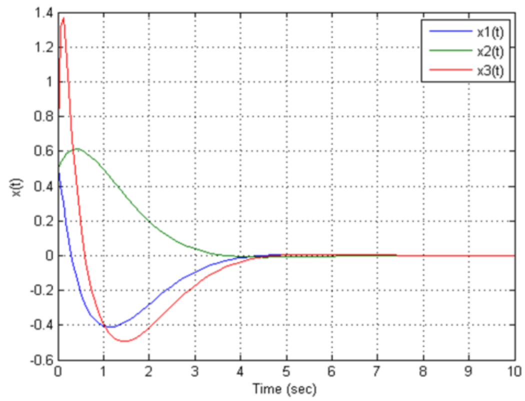

Figure 1. System state synchronization under proposed Fuzzy Sliding Mode Control.

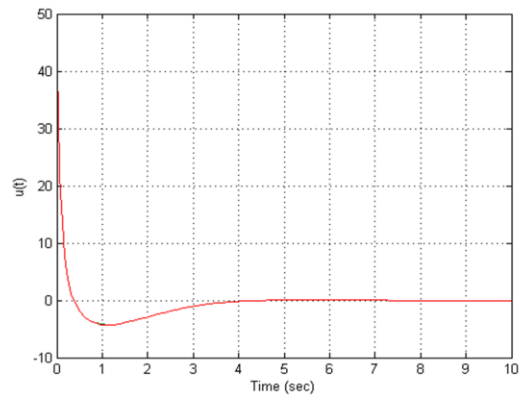

Figure 2. The proposed Fuzzy Sliding Mode Control.

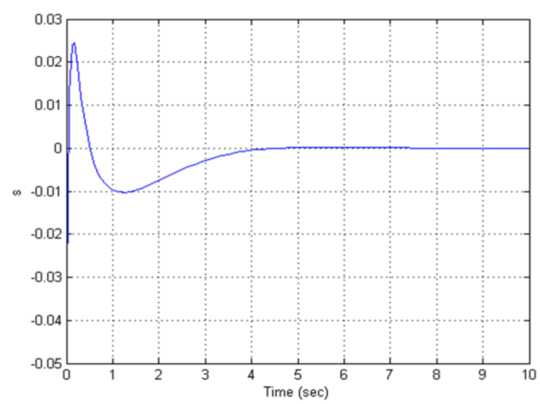

Figure 3. The sliding mode control of system.

V. Conclusion

In this paper we present the studied of fuzzy sliding mode control for a class chaotic nonlinear system. The approach design is conceptually simple thus reduce the conservatism and computational efforts even to complex nonlinear system. Furthermore, the analysis for system stability and controller design approach is formulated into Linear Matrix Inequality (LMI) terms. Finally, an illustrated example is used to show the feasibility and effectiveness of this control scheme.

REFERENCES

[1] T. Takagi and M. Sugeno, Fuzzy identification of systems and its application to modelling and control, IEEE Transactions on Systems, Man and Cybernetics, Vol.15, No.1, 1985, pp. 116-132.

[2] L. Zadeh, "Fuzzy sets," Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers,1996.

[3] K. Tanaka, T. Ikeda and H.O. Wang, Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs, IEEE Transactions on Fuzzy Systems, Vol.6, No.2, 1998, pp. 1-16.

[4] K. Tanaka and M. Sano, A robust stabilization problem of fuzzy control system and its application to backing up control of a truck-trailer, IEEE Transactions on Fuzzy Systems, Vol.2, No.2, 1994, pp. 119-134.

[5] T-M. Guerra and L. Vermeiren, Control laws for Takagi Sugeno fuzzy models, Fuzzy Sets & Systems, Vol.120, No.1, 2001, pp. 95-108.

[6] Park, C.-W.; Cho, Y.-W.; , "Robust fuzzy feedback linearisation controllers for Takagi-Sugeno fuzzy models with parametric uncertainties," *Control Theory & Applications, IET* , vol.1, no.5, pp.1242-1254, Sept. 2007

[7] T-M. Guerra and L. Vermeiren, LMI-based relaxed non quadratic stabilization conditions for non-linear systems in the Takagi-Sugeno form. Automatica, Vol.40, No.5, 2004, pp. 823-829

[8] W.L. Chiang, T.W. Chen, M.Y. Liu and C.J. Hsu, Application and robust H_∞ control of PDC fuzzy controller for nonlinear systems with external disturbance, Journal of Marine Science and Technology, Vol.9, No.2, 2001, pp. 84-90.

[9] C. W. Park, LMI-based robust stability analysis for fuzzy feedback linearization regulators with its application, Information sciences, vol 152, 2003, pp.287-301.

[10] S. Boyd, et al., Linear matrix inequalities in system and control theory, SIAM, Philadelphia, PA, 1994.

[11] Nachidi, M.; Benzaouia, A.; Tadeo, F.; Rami, M.A.; , "LMI-Based Approach for Output-Feedback Stabilization for Discrete-Time Takagi--Sugeno Systems," *Fuzzy Systems, IEEE Transactions on* , vol.16, no.5, Oct. 2008, pp.1188-1196.

[12] Xiao-Jun Ma; Zeng-Qi Sun; , "Output tracking and regulation of nonlinear system based on Takagi-Sugeno fuzzy model," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on* , vol.30, no.1, pp.47-59, Feb 2000

[13] W.C. Kim, S.C. Ahn and W.H. Kwon, Stability analysis and stabilization of fuzzy state space models, Fuzzy Set & Systems, Vol.71, No.1, 1995, pp. 131-142.

[14] K. Tanaka,T.Ikeda and H.O. Wang, Robust stability of a class of uncertain nonlinear system via fuzzy control: Quadratic stability, H-infiniti control theory and linear matrix inequalities., IEEE Transactions on Fuzzy Systems, Vol.4, No.1, Feb 1996, pp. 1-13.

[15] H.O. Wang, K. Tanaka and M.F. Griffin, An approach to fuzzy control of nonlinear system: stability and design issues, IEEE Transactions on Fuzzy Systems, Vol.4, No.1, Feb 1996, pp. 14-23.

[16] Pecora LM, Carroll TL Synchronization in chaotic systems. Phys Rev Lett 64: 1990, pp. 821–824.

[17] Chen G, Dong X, From Chaos to Order: Methodologies, Perspectives and Applications,World Scientific, Singapore 1998.

[18] Zhang HG, Huang W,Wang ZL, Chai TY, Adaptive synchronization between different chaotic systems with unknown parameters. Phys Lett A 350: 2006, pp.363–366.

[19] Zhang HG, Xie YH, Liu D, Synchronization of a class of delayed chaotic neural networks with fully unknown parameters. Dyn Contin Discrete Impuls Syst B 13: 2006 , pp. 297–308.

[20] Utkin, V.I,Variable structure systems with sliding mode, IEEE Transaction on Automatic Control, Vol.AC-22, No.2, April 1977, pp 212-222.

[21] Slotine, J.E and Li, W.P., Applied nonlinear control, Prentice Hall, Eaglewood Cliffs, NJ.

[22] K. D. Young, V. I. Utkin, and Ü. Özgüner, "A control engineer's guide to sliding mode control," IEEE Trans. Control Sys. Tech., vol. 7, 1999,pp. 328-342.

[23] Y. J. Huang and H. K. Wei, "Sliding mode control design for discrete multivariable systems with time-delayed input signals," Intern. J. Syst.Sci., vol. 33, 2002, pp. 789-798

[24] Chern T.L and Wu Y. C. Design of brushless DC position servo systems using integral variable structure approach. Proceeding of IEEE Part B, 140, 1993, pp.27-34.